Chapter 39
Study on Signal-In-Space Errors Calculation Method and Statistical Characterization of BeiDou Navigation Satellite System

Liang Chen, Wenhai Jiao, Xiaorui Huang, Changjiang Geng, Lun Ai, Lu Lu and Zhigang Hu

Abstract BeiDou Navigation Satellite System (BDS), currently including valid 14 in-orbit satellites-5GEOs, 5IGSOs and 4MEOs, had provided formal operation service in the late of 2012. Different type BeiDou satellite's Signal-In-Space Range Errors (SISRE) calculation formulation is derived in this paper according to navigation satellite's Signal-In-Space Range Errors definition. SISRE of Different type BeiDou satellites are assessed and analyzed on the basis of this SISRE calculation formulation by using broadcast ephemeris data and precise orbit products from January to October, 2012 (day of year 001–303, 2012). The results show: the broadcast ephemeris orbit accuracy of BeiDou system non-GEO satellites is better than GEO satellites; the accuracy of BeiDou satellites broadcast ephemeris orbit errors in radial direction, GEO is better than 1.5 m and non-GEO is better than 1.0 m, is the best; the average of GEO satellites rms Signal-In-Space Range errors including orbit errors only is better than 2.0 m, while non-GEOs is better than 0.8 m.

Keywords BeiDou system · SISRE · Formula derivation · Signal-in-space · Accuracy statistical characterization

L. Chen (✉) · W. Jiao · C. Geng · L. Lu
Test and Assessment Research Center, China Satellite Navigation Office, 1 Fengyingdong Road, Beijing 100094, China
e-mail: sdkdchenliang@163.com; chenliang880905@163.com

L. Chen · C. Geng · L. Lu
GNSS System Engineering Center, China Academy of Aerospace Electronics Technology, 1 Fengyingdong Road, Beijing 100094, China

X. Huang · L. Ai
Beijing Research Institute of Telemetry, Nandahongmen Road, Beijing 100076, China

Z. Hu
GNSS Research Center, Wuhan University, 129 Luoyu Road, Wuhan 430079, China

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39.1 Introduction

BeiDou Navigation Satellite System developed and operated by China independently, can provide high-precision and high-reliability Position, Navigation and Timing service (PNT) and will be completed to be BeiDou-II, which include 35 satellites and can overlap the earth surface in 2020s [1, 2]. BeiDou Navigation Satellite System had provided trial service for Asia-pacific area users in the late of 2011. The event that 16th satellite was successfully launched on October 25, 2012 respected that the second-stage satellite network had completed, which will further promote and improve the BeiDou system service performance immensely. With the ICD formal version publishing, BeiDou had provided formal operation service for Asia–pacific area users in Dec 27, 2012.

BeiDou Navigation Satellite System currently includes valid 14 in-orbit working satellites-5 Geostationary Orbit satellites (GEOs), 5 inclined Geo-synchronization orbit satellites (IGSOs) and 4 Medium Earth Orbit Satellites (MEOs). The Fig. 39.1 shows the BeiDou system footprints and coverage (reference time–0:00:00 November 11, 2012). As can be seen from the figure, the number of visible BeiDou satellites is more than 8 in the area of Asia-pacific. The satellite marked red is GEO (C02), which was launched on October 25 and is in on-orbit tests now; The GEO (C03) satellite is adjusting to 110.5°E.

Currently, Signal-In-Space Ranging Error (SISRE) is one of key indicators which can assess the system service performance of navigation satellite system effectively. Different type BeiDou satellite's Signal-In-Space Range Errors (SIS-RE) calculation formula is derived in this paper according to navigation satellite's Signal-In-Space Range Errors definition. Broadcast orbit and Signal-In-Space accuracy of BeiDou system are analyzed by those calculation formulae and a series of results and conclusions are got in this paper too.

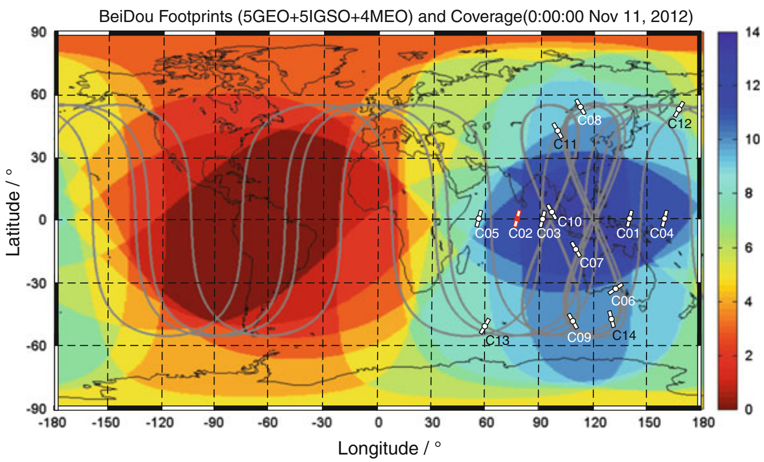


Fig. 39.1 BeiDou satellites footprints and depth of coverage (reference time–0:00:00 2012-11-11)

39.2 Calculation Method of BeiDou Signal-In-Space Errors

39.2.1 Navigation Satellite SISRE

The Signal-In-Space Ranging Error (SISRE) is a key performance indicator for assessing the performance of the BeiDou Navigation Satellite System. SISRE can be defined as the difference between the satellite position based on the broadcast navigation data (position and clock) and the truth, projected on the line-of-sight to the user [3]. The instantaneous SISRE values can be evaluated at a large number of spatial points spread evenly across the satellite's coverage on the surface of Earth, and the global average SISRE can be computed as the rms of the instantaneous SISRE value at each of those spatial points [4].

As shown in Fig. 39.2, R_S is orbital radius of navigation satellite, r_E is the average radius of the Earth; $\overrightarrow{E} = (-C,\ A,\ -R)$ is the deviation vector of broadcast orbit error in the Cross, Along, Radial direction, respectively. A space Cartesian coordinate system is created as shown in the figure. A point D, the projection of which in xoy plane is D', is selected arbitrarily in the satellite S coverage on the earth surface. Assuming the angle between OD and the z-axis is α, OD' and x-axis β. From those, $\overrightarrow{SD}$ direction normalized vector can be got as follow:

$$\overrightarrow{l} = \frac{\overrightarrow{SD}}{\left|\overrightarrow{SD}\right|} = \frac{(r_E \sin\alpha \cos\beta,\ r_E \sin\alpha \sin\beta,\ r_E \cos\alpha - R_S)}{\sqrt{r_E^2 - 2 r_E R_S \cos\alpha + R_S^2}} \tag{39.1}$$

If we assume uniform distribution of positions on the surface of Earth [5, 6], for the point $D\ (\alpha,\ \beta)$ on the surface of Earth the superficial area in the range of $[\alpha,\ \alpha + \mathrm{d}\alpha]$, $[\beta,\ \beta + \mathrm{d}\beta]$ is $[\beta,\ \beta + \mathrm{d}\beta]$, then the superficial area of satellite S coverage on the surface of Earth is $\int_0^{2\pi} \int_0^{\frac{\pi}{2} - \theta} r_E^2 \cdot \sin\alpha\, \mathrm{d}\alpha\, \mathrm{d}\beta$. So the joint probability distribution of the point in satellite coverage on the surface of Earth is:

$$p(\alpha,\ \beta) = \frac{r_E^2 \cdot \sin\alpha\, \mathrm{d}\alpha\, \mathrm{d}\beta}{\int_0^{2\pi} \int_0^{\frac{\pi}{2} - \theta} r_E^2 \cdot \sin\alpha\, \mathrm{d}\alpha\, \mathrm{d}\beta} = \frac{\sin\alpha\, \mathrm{d}\alpha\, \mathrm{d}\beta}{2\pi(1 - \sin\theta)} \tag{39.2}$$

According to the definition of "Global Positioning System Standard Positioning Service Performance Standard 2008" for resolving the global average SISRE formula, the global average SISRE is RMS statistical value of the projection error of satellite signal-in-space error (orbit error $\overrightarrow{E}$ and satellite clock error T) on the signal propagation direction $\overrightarrow{l}$ [6, 7]. From Fig. 39.2, variable range of α is $\left[0,\ \frac{\pi}{2} - \theta\right]$, and the β is $[0,\ 2\pi]$. Thus, the RMS formula of SISRE is as follows:

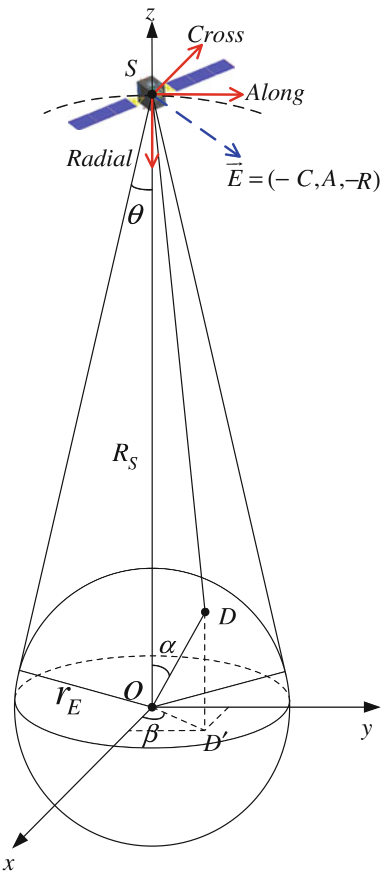


Fig. 39.2 Diagram of orbit errors in *Radial*, *Along*, *Cross* direction and Signal-In-Space range errors formula derivation

$$rms\,SISRE = \sqrt{\int_0^{2\pi}\int_0^{\frac{\pi}{2}-\theta}\left[\vec{E}\cdot\vec{l} - c\cdot T\right]^2 \sin\alpha\, d\alpha\, d\beta \Big/ 2\pi(1 - \sin\theta)} \tag{39.3}$$

where, c is the speed of the light in vacuum, T is the satellite clock error.

39.2.2 SISRE Formulation of BeiDou System

BeiDou satellite navigation constellation consists of GEO satellites, IGSO satellites and MEO satellites. Because orbit height of different type BeiDou satellites is different with GPS, the SISRE calculation formula is different. In order to get the BeiDou system SISRE approximate calculation formula, we assume that the Earth is a uniform sphere, average radius of which is 6,371 km; BeiDou GEO and IGSO satellite's average orbit height are 35,786 km, and the MEO's is 21,528 km [8, 9].

According to those assumptions, we can obtain the BeiDou GEO and IGSO satellite's $\theta = 8.69°$, MEO satellite's $\theta = 13.21°$ in the Fig. 39.2. Substituting θ into the formula (39.3), different type BeiDou navigation satellite's SISRE approximate calculation formula can be obtained as follows:

$$\begin{aligned} rms\,SISRE_{GEO\&IGSO} &= \sqrt{0.9842 \cdot R^2 - 1.9841 \cdot R \cdot cT + (c \cdot T)^2 + 0.0079 \cdot (A^2 + C^2)} \\ &\approx \sqrt{(0.99 \cdot R - c \cdot T)^2 + \frac{1}{127} \cdot (A^2 + C^2)} \end{aligned} \tag{39.4}$$

$$\begin{aligned} rms\,SISRE_{MEO} &= \sqrt{0.9631 \cdot R^2 - 1.9627 \cdot R \cdot cT + (c \cdot T)^2 + 0.0185 \cdot (A^2 + C^2)} \\ &\approx \sqrt{(0.98 \cdot R - c \cdot T)^2 + \frac{1}{54} \cdot (A^2 + C^2)} \end{aligned} \tag{39.5}$$

The statistics results SISRE in this paper only consider the satellite's broadcast orbit error, that is:

$$rms\,SISRE_{G\&I-Orbit-error-only} = \sqrt{(0.99 \cdot R)^2 + \frac{1}{127} \cdot (A^2 + C^2)} \tag{39.6}$$

$$rms\,SISRE_{M-Orbit-error-only} = \sqrt{(0.98 \cdot R)^2 + \frac{1}{54} \cdot (A^2 + C^2)} \tag{39.7}$$

39.3 Data Sources

39.3.1 WHU's Precise Ephemeris

Some data used in this paper are based on the international GNSS monitoring and assessment system (iGMAS). As the Data and Analysis Center of the iGMAS, Wuhan University (WHU) set out to establish the BeiDou Experimental Tracking Stations (BETS, as shown in Fig. 39.3) worldwide at the beginning of 2011 to carry out comprehensive scientific and application studies about the BeiDou Navigation Satellite System. Currently, a continuous tracking network for BeiDou in-orbit satellites has been basically formed. Currently, the BETS network consist of nine domestic stations and seven overseas continuous tracking stations, which are equipped with UR240-CORS, independently developed by China and supporting Beidou/GPS dual system quadruple frequency high-accuracy receivers. By PANDA (Position And Navigation Data Analysis) software processing the observations of BETS network, BeiDou system precise orbit products can be obtained in format sp3. From the orbit results, the radial accuracy of the BeiDou satellite precise orbit is better than 10 cm, the other directions accuracy of

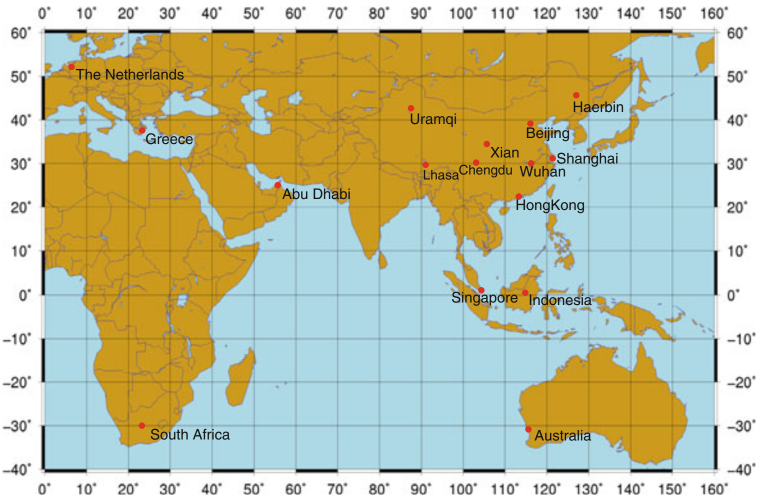


Fig. 39.3 BeiDou experimental tracking stations (BETS) for producing precise ephemeris products

overlapping arcs can reach to 10–20 cm [10]. Relative to the meters accuracy of BeiDou broadcast ephemeris orbit, the precise ephemeris products can be used as its reference true value absolutely.

What need to mention is that the precise orbit accuracy of IGSO satellite is better than GEO satellite's from the orbit results. Some reasons are as follow: firstly, because flare angle of the GEO satellite relative to the earth is very small (less than 18°) and BETS network belongs to a local distribution tracking network currently, observation geometry structure strength to the GEO satellites is poor. Secondly, the GEO satellite is a kind of Geostationary Orbit satellite and keep stationary relative to the ground tracking stations, the geometry changes of satellite-to-station are very small, the information increased by increasing the observation time is limited, so some system errors (such as clock error and tracking station position deviation) are hard to be resolved and separated, which makes the GEO satellite orbit errors in along track direction are large. These factors may affect the statistical analysis result of BeiDou broadcast ephemeris orbit accuracy in the following paper.

39.3.2 Data Selection and Processing Strategies

In order to assess and analyse the accuracy of the BeiDou system signal-in-space effectively, consecutive ten months broadcast ephemeris data are selected since BeiDou system began to provide trial service in the late of last year (Day of Year 001-303, 2012). The selected satellite data arcs are shown in Table 39.1. Taking

Table 39.1 Data arc selected in this paper

Satellite no.	Satellite type	Start time	End time
C01	GEO	2012.1.1	2012.10.29
C02	GEO	–	–
C03	GEO	2012.1.1	2012.10.29
C04	GEO	2012.1.1	2012.10.29
C05	GEO	2012.5.1	2012.10.29
C06	IGSO	2012.1.1	2012.10.29
C07	IGSO	2012.1.1	2012.10.29
C08	IGSO	2012.1.1	2012.10.29
C09	IGSO	2012.1.1	2012.10.29
C10	IGSO	2012.1.1	2012.10.29
C11	MEO	2012.7.1	2012.10.29
C12	MEO	2012.7.1	2012.10.29
C13	MEO	–	–
C14	MEO	–	–

the BeiDou precise ephemeris provided by iGMAS Data and Analysis Center (WHU) as the reference true value and following the BeiDou SISRE calculation formula obtained above, BeiDou system present broadcast ephemeris orbit accuracy and SISRE are assessed in the follow paper.

39.4 Statistical Characterization of BeiDou Signal-In-Space Errors

39.4.1 Statistics and Analysis of Broadcast Ephemeris Orbit Error

In order to analyze the change rules of different type BeiDou satellite's broadcast orbit error in each direction over time, GEO (C04), IGSO (C06) and MEO (C11) orbit data in September, 2012 (Day of Year 245–274) are selected to give typical examples in this part, as shown in Fig. 39.4.

At the same time, we also statistic the mean and standard deviation of BeiDou in-orbit working satellite's ephemeris errors in Radial, Along and Cross direction from January to October, 2012, which is shown in Fig. 39.5, and the RMS of BeiDou ephemeris in each direction according to the formula (39.8), as shown in Table 39.2.

$$RMS = \sqrt{\sum_{i=1}^{n} v_i \cdot v_i \Big/ n - 1} \tag{39.8}$$

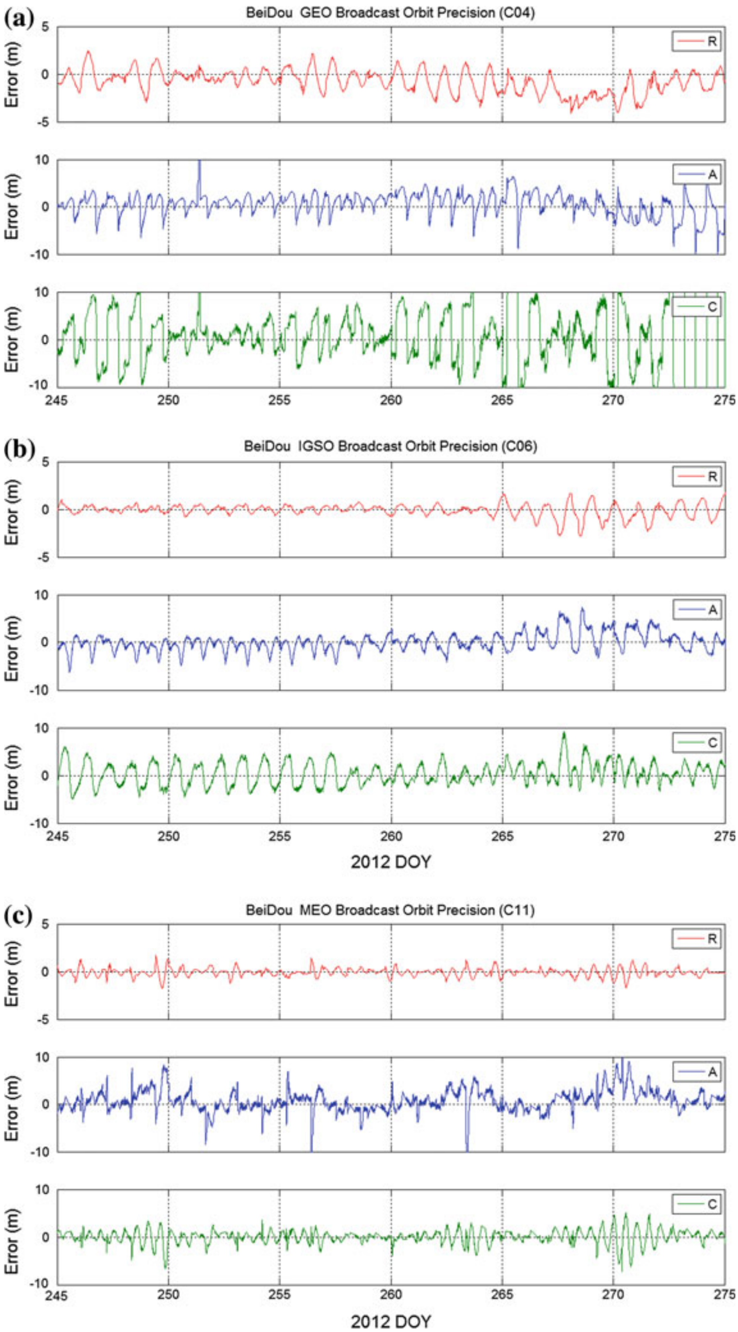


Fig. 39.4 Broadcast ephemeris errors change curve versus time of Different type BeiDou satellites are BeiDou MEO Broadcast Orbit Precision **a** C04, **b** C06, and **c** C11

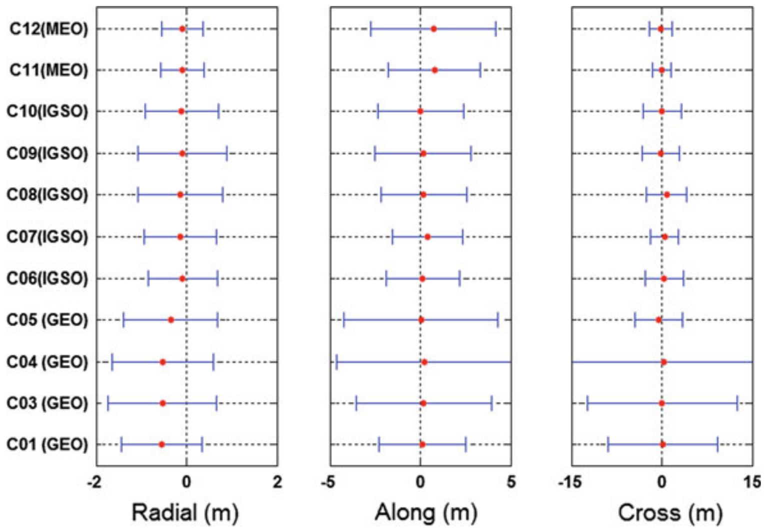


Fig. 39.5 Mean and once standard deviation of BeiDou Navigation Satellite System broadcast ephemeris errors

Table 39.2 RMS Accuracy statistic of BeiDou satellites broadcast orbit errors in Radial, along and cross direction

Satellite no.	Satellite type	Radial (m)	Along (m)	Cross (m)
C01	GEO	1.041	2.417	9.063
C03	GEO	1.310	3.744	12.471
C04	GEO	1.231	4.867	20.434
C05	GEO	1.109	4.238	3.935
C06	IGSO	0.774	2.051	3.191
C07	IGSO	0.807	1.972	2.342
C08	IGSO	0.953	2.372	3.341
C09	IGSO	0.991	2.648	3.132
C10	IGSO	0.817	2.384	3.172
C11	MEO	0.492	2.654	1.519
C12	MEO	0.470	3.540	1.914

Combining Figs. 39.4, 39.5 and Table 39.2, what can be obtained as follow:

1. The broadcast orbit errors in each direction exit significant periodic fluctuation. Figure 39.4 shows that there are obvious fluctuations in all directions since DOY265.
2. Although ephemeris errors are generally assumed to be zero-mean, the reality may be different. Figure 39.5 indicates the mean of Radial, Along and Cross normalized by the standard deviation. In Fig. 39.5, the red dots denote the mean, while the blue lines with a length of once the standard deviation are centered at the red dots. From the figure, we can get that most of the mean of

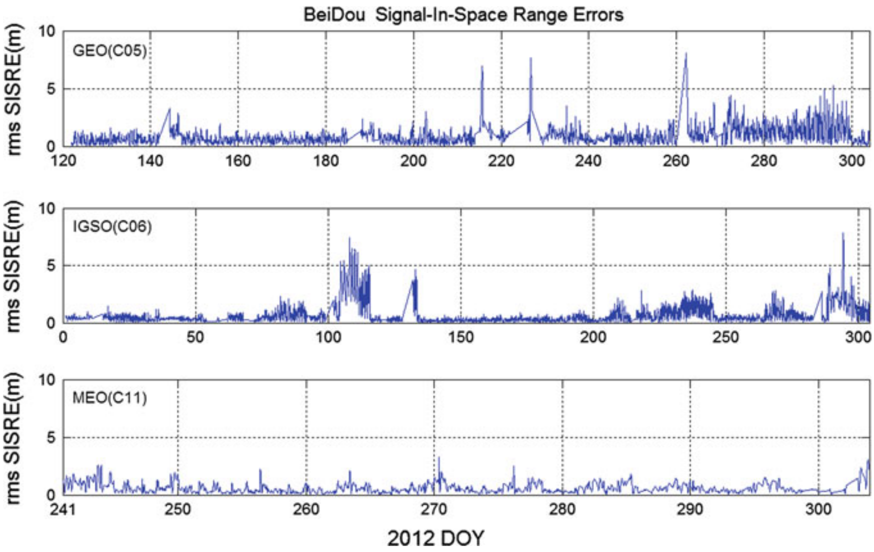
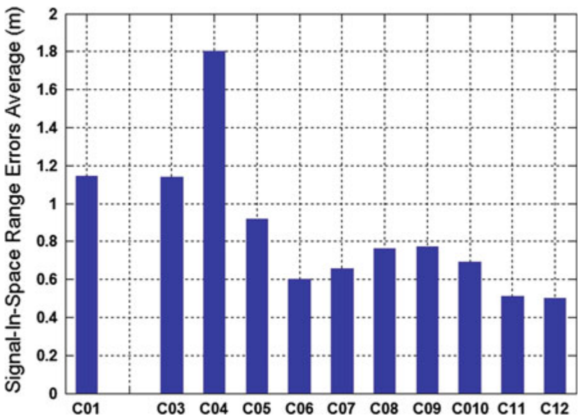


Fig. 39.6 Orbit-error-only SISRE change curve versus time of different type BeiDou satellites

Fig. 39.7 Mean of BeiDou system Orbit-error-only SISRE from DOY 001-303, 2012



BeiDou non-GEOs ephemeris errors in each direction are near zero, while radialtrack errors usually have a negative mean.

3. Table 39.2 indicates the accuracy of radial ephemeris errors is the best, with GEOs is better than 1.5 m, IGSOs precedes 1.0 m and MEOs is superior to 0.5 m. This is because the ground tracking stations to satellite observations changes in the radial is more sensitive than the other two directions, and the force model in along track and cross track is not perfect.
4. Figure 39.5 and Table 39.2 indicate the ephemeris errors accuracy of IGSOs and MEOs in each direction is better than GEOs'. In Fig. 39.5, there is a larger standard deviation of the C04 (GEO) ephemeris error in cross direction, which is 20.43 m.

39.4.2 SISRE Statistics and Analysis

By the formula (39.6, 39.7), what can be obtained is that radial errors have a greater impact on SISRE; better radial error can obtain better SISRE. Using the data in Table 39.1, we calculate and analyze the orbit-error-only SISRE (SISRE $_{\text{Orbit-error-only}}$) of different type BeiDou satellites. Figure 39.6 shows orbit-error-only SISRE change curve versus time of Different type BeiDou satellites, taking GEO(C05), IGSO(C06) and MEO(C11) as typical examples.

As can be seen from the Fig. 39.5, the orbit-error-only SISRE changes of different type BeiDou satellites are smooth; Most of GEO and IGSO satellite's orbit-error-only SISRE is better than 3.0 m, while MEO satellite's is superior to 2.0 m.

Figure 39.7 shows the statistic mean of BeiDou satellites orbit-error-only SISRE from DOY 001-303, 2012. From that, the mean of GEOs orbit-error-only SISRE, better than 2.0 m, is larger than non-GEO satellite's, which is superior to 1.8 m. From the statistical results, the BeiDou system's orbit-error-only SISRE has keep the same level with GLONASS [5].

39.5 Conclusions

Now BeiDou Navigation Satellite System has completed the work of second-stage satellite network (5GEO+5IGSO+4MEO). Based on the characteristics of the BeiDou satellite orbit, Different type BeiDou satellite's Signal-In-Space Range Errors (SISRE) calculation formula, as the key, is derived in this paper. Using the broadcast ephemeris data and precise ephemeris product from January to October, 2012 (Day of Year 001-303, 2012), broadcast orbit accuracy and Signal-In-Space performance of BeiDou system are elevated and analyzed by those calculation formulae. Statistical analysis results show as follow:

1. The broadcast ephemeris orbit accuracy of BeiDou system non-GEO satellite is better than GEO satellite;
2. The radial ephemeris error accuracy of each BeiDou satellites is the best, GEO is better than 1.5 m, non-GEO is superior to 1.0 m;
3. The mean of GEO satellite's Orbit-error-only SISRE is better than 2.0 m, while non-GEOs' is superior to 0.8 m.
4. BeiDou system broadcast ephemeris orbit-error-only SISRE has keep the same level with GLONASS. But comparing to GPS system's better than 1.0 m broadcast ephemeris orbit-error-only SISRE [11], there is still room for further improvement for the BeiDou system's broadcast ephemeris orbit accuracy.

With the further development of the BeiDou Navigation Satellite System, the broadcast ephemeris orbit accuracy will be further improved and enhanced to provide better positioning and navigation services for global users

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